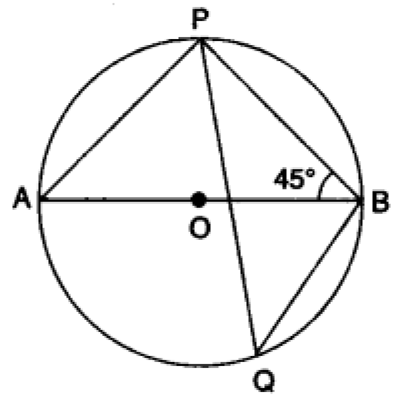
Circles

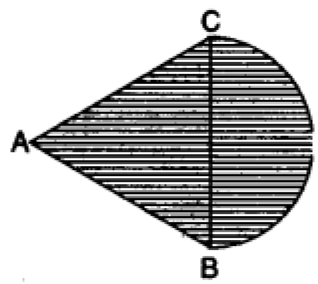
1. In the given figure, O is the centre of the circle and ∠PBA = 45°. Calculate the value of ∠PQB.



(2007)

Answer:
∠AOB = 180°
⇒ ∠APB = 90° (angle of diameter)
⇒ ∠PAB = 90° – 45° = 45°
⇒ ∠PQB = 45° (angle for same arc)

2. In an equilateral Δ ABC of side 14 cm, side BC is the diameter of a semi-circle as shown in the figure below. Find the area of the shaded region. [3]
(take π = 22/7 and y (√3 = 1.732)



(2007)

Answer:

(b) Area of equilateral triangle $ABC = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times (14)^2 = \frac{\sqrt{3}}{4} \times 196$

$= 49\sqrt{3}\ \text{cm}^2$

$= 84{\cdot}868\ \text{cm}^2$

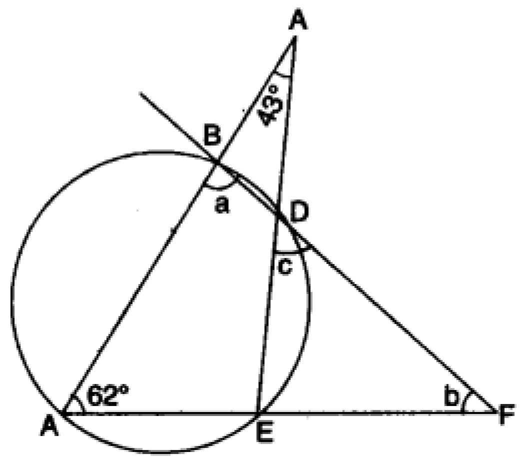
Area of semi-circle $= \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7^2$

$= 77\ \text{cm}^2$

Total area of shaded region $= 84{\cdot}868 + 77$

$= 161{\cdot}868\ \text{cm}^2$.

3. In the given figure, if ∠ACE = 43° and ∠CAF = 62° find the values of a, b and c.



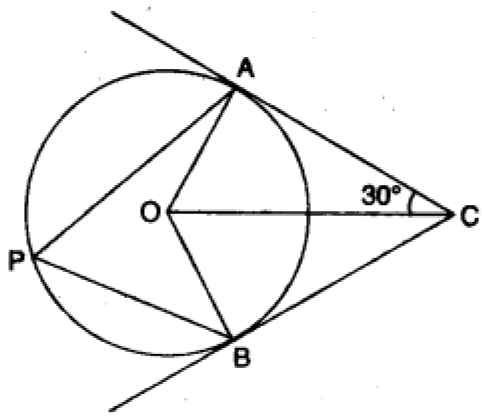
(2007)

Answer:

$$\begin{aligned}
\angle AEC &= 180° - (\angle EAC + \angle ACE) \\
&= 180° - (62° + 43°) = 180° - 105° \\
&= 75° \\
\angle CEF &= 180° - 75° \quad \text{(cyclic quadrilateral)} \\
&= 105° \\
\angle ABD = \angle a &= 180° - 75° \\
&= 105° \\
\angle b = \angle AFD &= 180° - (62° + 105°) \\
&= 180° - (167°) \\
&= 13° \\
\angle c = \angle EDF &= 180° - (105° + 13°) \\
&= 180° - (118°) \\
&= 62°
\end{aligned}$$

4. In the given figure O is the centre of the circle. Tangents at A and B meet at C. If ∠AOC = 30°, find

(i) ∠BCO
(ii) ∠AOB
(iii) ∠APB

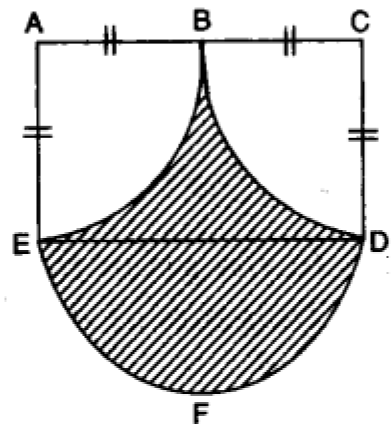


(2011)

Answer:

		$\Delta ACO \cong \Delta BCO$	(R.H.S.)
∴		$\angle BCO = \angle ACO$	(C.P.C.T.)
(i)		$\angle BCO = 30°$	
	In Δ ACO,	$\angle OAC = 90°$	(Radius is perpendicular to tangent)
	∴	$\angle AOC = 60°$	
	Also	$\angle BOC = 60°$	(C.P.C.T.)
(ii)		$\angle AOB = 120°$	
(iii)		$\angle APB = 60°$	(Angle at circumference is half the angle at the centre)

5. Calculate the area of the shaded region, if the diameter of the semi-circle is equal to 14 cm. Take π = 22/7



(2011)

Answer:

Area of shaded portion = Complete area – area of the two quadrants

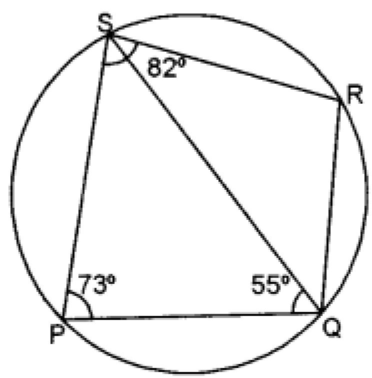
= (Area of ACDE + Area of semi circle EFD) – (Area of Quadrant ABE + Area of Quadrant BCD)

$$= \left\{14 \times 7 + \frac{\pi}{2}(7)^2\right\} - \left\{\frac{\pi}{4}(7)^2 + \frac{\pi}{4}(7)^2\right\}$$

$$= \left\{14 \times 7 + \frac{\pi}{2}(7)^2\right\} - \left\{\frac{\pi}{2}(7)^2\right\}$$

$= 98\ \text{cm}^2$.

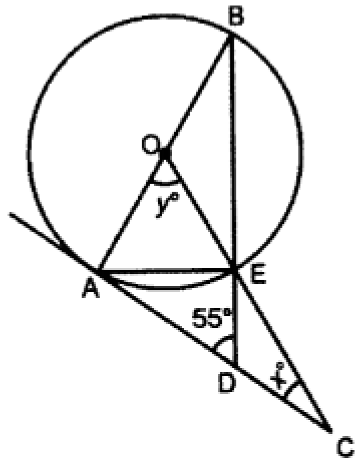
6. PQRS is a cyclic quadrilateral. Given ∠QPS = 73°, ∠PQS = 55° and ∠PSR = 82°, calculate:
(i) ∠QRS
(ii) ∠RQS
(iii) ∠PRQ



(2018)

Answer:
(i) Since PQRS is a cyclic quadrilateral
∠QPS + ∠QRS – 180°
⇒ 73° + ∠QRS = 180°
⇒ ∠QRS = 180° – 73°
∠QRS = 107°
(ii) Again, ∠PQR + ∠PSR = 180°
∠PQS + ∠RQS + ∠PSR = 180°
55° – ∠RQS + 82° = 180°
∠RQS = 180° – 82° – 55° = 43°
(iii) In ΔPQS, by using angles sum property of a Δ.
∠PSQ + ∠SQP + ∠QPS = 180°
∠PSQ + 55° + 73° = 180°
∠PSQ = 180° – 55° – 73°
∠PSQ = 52°
Now, ∠PRQ = ∠PSQ = 52° [Oop. ∠s of the same segment]
Hence, ∠QRS = 107°, ∠RQS = 43° and
∠PRQ = 52°

7. In the given figure, AC is a tangent to the circle with centre 0.
If ∠ADB = 55°, find x and y. Give reasons for your answers.



(2019)

Answer:
We know that angle between the radius and the tangent at the point of contact is right angle.

$\therefore \quad \angle A = 90°$

Also, in ΔOBE, $OB = OE =$ radius (r)

$\therefore \quad \angle B = \angle OEB \quad \ldots(i)$

In ΔABD,

$$\angle A + \angle B + \angle ADB = 180°$$
$$90° + \angle B + 55° = 180°$$
$$\angle B = 180° - 90° - 55° = 35°$$

Thus, $\angle B = \angle OEB = 35°$

$\angle OEB = \angle DEC = 35°$ [vertically opp. $\angle$s]

$$\angle EDC + \angle ADE = 180°$$
$$\angle EDC + 55° = 180°$$

$\Rightarrow \quad \angle EDC = 180° - 55° = 125°$

In ΔEDC,

$$\angle DEC + \angle EDC + x = 180°$$
$$35° + 125° + x = 180°$$

$\Rightarrow \quad x = 180° - 35° - 125° = 20°$

In ΔAOC,

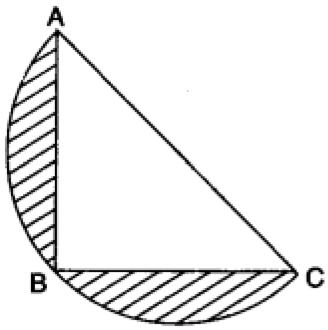
$$\angle A + y° + x° = 180°$$

$\Rightarrow \quad 90° + y° + 20° = 180°$

$\Rightarrow \quad y° = 180° - 90° - 20° = 70°$

Hence, $x = 20°$ and $y = 70°$.

8. ABC is an isosceles right angled triangle with ∠ABC = 90°. A semicircle is drawn with AC as the diameter. If AB = BC = 7 cm, find the area of the shaded region. (Take n = 22/7)



(2012)

Answer:

Let ABC is a right angled triangle. So

$$AC^2 = AB^2 + BC^2$$

$$= (7)^2 + (7)^2 = 2(7)^2$$

$$AC = 7\sqrt{2}$$

$$\text{Area of semi circle} = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7\sqrt{2}}{2}\right)^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times \frac{49 \times 2}{4} = 38{\cdot}5 \text{ cm}^2$$

$$\text{Area of } \Delta\, ABC = \frac{1}{2} \times 7 \times 7 = 24{\cdot}5 \text{ cm}^2$$

$$\therefore \quad \text{Area of shaded region} = \text{Area of semi circle} - \text{Area of } \Delta\, ABC.$$

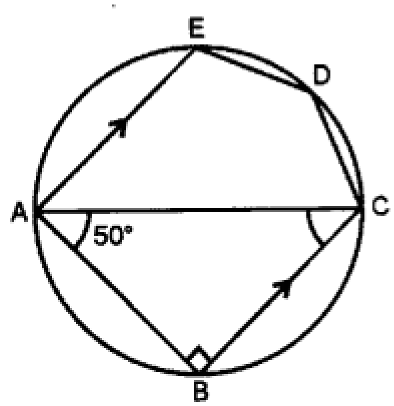
$$= 38{\cdot}5 - 24{\cdot}5 = 14 \text{ cm}^2.$$

9. In the given figure, ABCDE is a pentagon inscribed in a circle such that AC is a diameter and side BC||AE. If ΔBAC = 50°, find giving reasons :

(i) ∠ACB

(ii) ∠EDC

(iii) ∠BEC

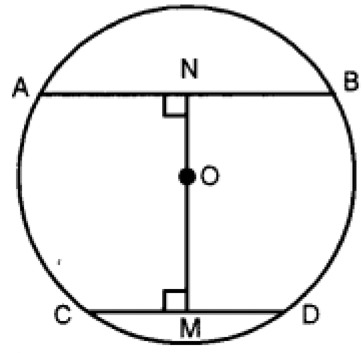


Hence, prove that BE is also a diameter.

(2019)

Answer:
Since AC is a diameter and angle in a semi-circle is right angle
∠B = 90° and
∠ACB = 40°
Also, BC || AE
∠EAC = ∠ACB
= 40°
[alt. int. ∠s]
In cyclic quadrilateral ACDE
∠EAC + ∠EDC = 180°
40° + ∠EDC = 180°
∠EDC = 180° – 40° = 140°
∠BEC = ∠BAC
= 50° [∠s in the same segment]
Also,
∠EAC = ∠EBC
= 40° [∠s in the same segment]
∠ABE = ∠ABC – ∠EBC
= 90° – 40°
= 50°
Again, ∠ABE = ∠ACE = 50° [∠s in the same segment]
Now, ∠ACE +∠ACB = 50° + 40°
= 90°
∠BCE = 90°
Hence, BE is a diameter, because angle is a semi-circle is right angle.

10. AB and CD are two parallel chords of a circle such that AB = 24 cm and CD = 10 cm. If the radius of the circle is 13 cm, find the distance between the two chords.



(2017)

Answer:

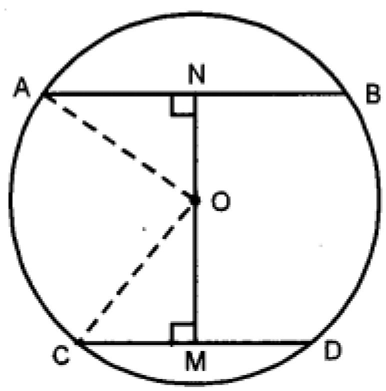
Here, O is the centre of the given circle of radius 13 cm. AB and CD are two parallel chords, such that AB = 24 cm and CD = lo cm.

Join OA and OC.

Since ON⊥AB and 0M ⊥CD.

∴ M, O and N are collinear and M, N are mid-points of CD and AB.

Now, in rt. ∠ed ΔANO, we have



ON2 = 02 AN2

= 132 – 122

= 169 – 144 = 25

ON=$\sqrt{25}$ = 5cm

Similarly, in it. ∠ed ΔCMO, we have

OM2 = OC2 – CM2

= 132 – 52

= 169 – 25 = 144

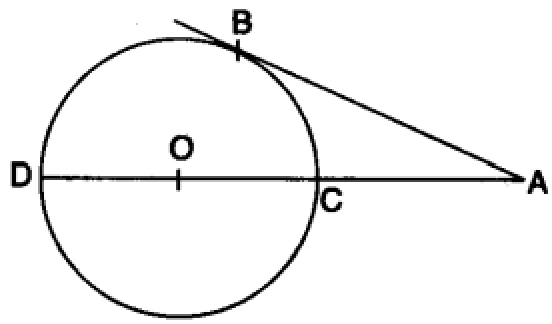
0M = $\sqrt{144}$= 12cm

Hence, distance between the two chords

NM = NO + OM

= 5 + 12
= 17 cm

11. In the given figure O is the centre of the circle and AB is a tangent at B. If AB = 15 cm and AC = 7.5 cm. Calculate the radius of the circle.



(2012)

Answer:

Applying intercept theorem

$$AC \times AD = AB^2$$

$$7{\cdot}5 \times (7{\cdot}5 + 2R) = 15^2$$

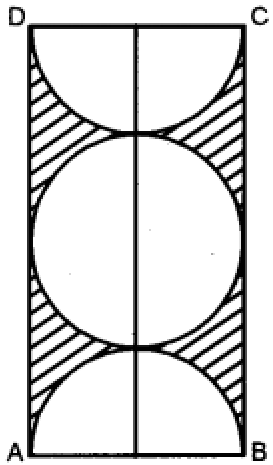
where R is the radius of the circle

$$(7{\cdot}5 + 2R) = \frac{15 \times 15}{7{\cdot}5} = 30$$

$$2R = 22{\cdot}5$$

$$\Rightarrow \quad R = 11{\cdot}25 \text{ cm.}$$

12. In the given figure ABCD is a rectangle. It consists of a circle and two semi-circles each of which are of radius 5 cm. Find the area of the shaded region. Give your answer correct to three significant figures.



(2017)

Answer:
Here, radius of a circle and two semi-circles = 5 cm
Length of the rectangle = 5 + 10 + 5 = 20 cm
Breadth of the rectangle = 10 cm
Now, area of the shaded part = Area of rectangle – 2 × Area of circle

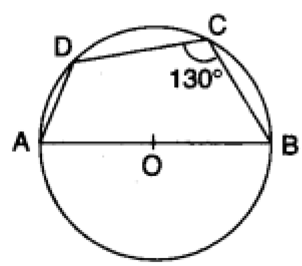
$$= 20 \times 10 - 2 \times \frac{22}{7} \times 5 \times 5$$

$$= 200 - \frac{1100}{7} = 200 - 157.14$$

$$= 42.86 \text{ cm}^2 \text{ or } 42.9 \text{ cm}^2.$$

13.

(a) *In the given figure, AB is the diameter of a circle with centre O.*
$\angle BCD = 130°$. Find :
(i) $\angle DAB$, (ii) $\angle DBA$ **[3]**



(b) *Given* $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$. *Write :*

(i) *the order of the matrix X*

(ii) *the matrix X.* **[3]**

(2012)

Answer:
(a)

On joining BD, $\angle$ ADB is in the semicircle.

$$\angle ADB = 90°$$

(Angle in a simicircle is right angle)

(i) Let ABCD is a cyclic quadilateral.

$$\therefore \quad \angle BCD + \angle DAB = 180°$$
$$130° + \angle DAB = 180°$$
$$\angle DAB = 180° - 130° = 50°$$

(ii) Now, $\angle BAD + \angle BDA + \angle DBA = 180°$

$$90° + 50° + \angle DBA = 180°$$
$$\angle DBA = 40°$$

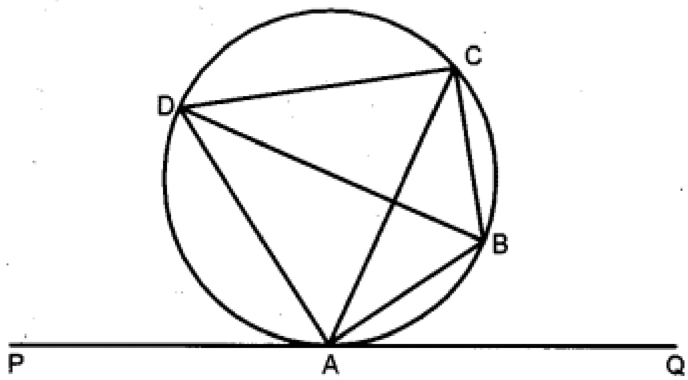
(b)

$$\because \quad \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$
$$\begin{bmatrix} 2a + b \\ -3a + 4b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$
$$2a + b = 7 \quad \text{......(1)}$$
$$-3a + 4b = 6 \quad \text{......(2)}$$

Solving (1) and (2), we get

$$a = 2, \ b = 3$$
$$\therefore \quad X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

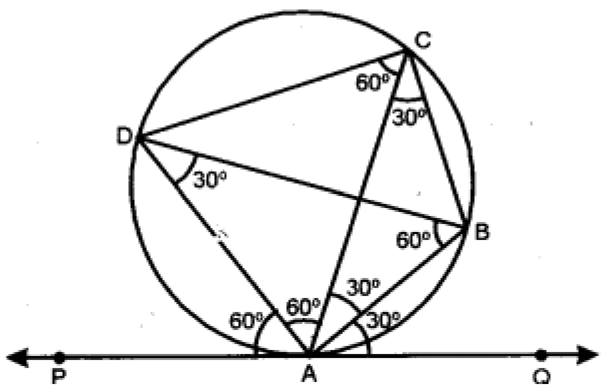
14. In the given figure PQ is a tangent to the circle at A. AB and AD are bisectors of $\angle$CAQ and $\angle$PAC. If $\angle$BAQ = 30° prove that :
(i) BD is a diameter of the circle.
(ii) ABC is an isosceles triangle.



(2017)

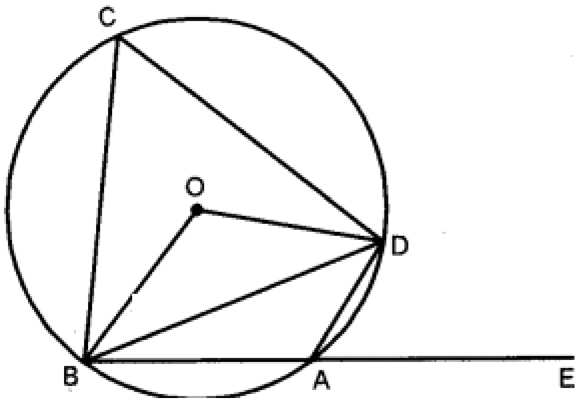
Answer:

Here PQ, is a tangent of the circle at A .



∠BAQ = 300. Since AB is
angle bisector of ∠CAQ.
∴ ∠CAB = ∠BAQ = 300
Again, ∠PAC = 180° – ∠CAQ
= 180°- 30°- 30°
= 120°
Also, AD is angle bisector of ∠PAC
∴ ∠PAD = ∠CAD = 600
Since angles in the corresponding alternate segment are equal
∴ ∠ADB =∠BAQ = 300 and ∠DBA = ∠PAD = 60°
Also, angles in same segment are equal
∴ ∠DCA = ∠DBA = 600
and ∠ACB = ∠ADB = 30°
Now, ∠DCB = ∠DCA + ∠ACB = 600 + 300 = 90°
We know that angle in a semi-circle is right angle.
Thus, BD is a diameter of the circle.
In ΔACB, ∠ACB = ∠CAB = 30°
Hence, ΔABC is an isosceles triangle.

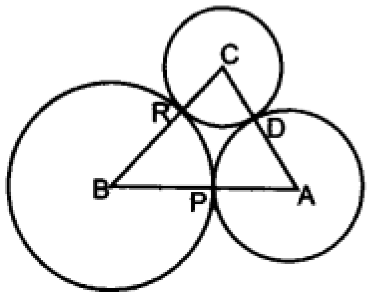
15. In the figure given, O is the centre of the circle. LDAE = 700.
Find giving suitable reasons, the measure of:
(i) ∠BCD
(ii) ∠BOD
(iii) ∠OBD



(2017)

Ans.
Here, ∠DAE = 70°
∴ ∠BAD = 180° ∠DAE
[a linear pair]
= 180° – 70° = 110°
ABCD is a cyclic quadrilateral
∴ ∠BCD + ∠BAD = 180°
∠BCD + 110° = 180°
⇒ ∠BCD = 180° – 110°
= 70°
Since angle subtended by an arc at the centre of a circle is twice the angle subtended at the remaining part of the circle.
∴ ∠BOD = 2∠BCD = 2 × 70° = 140°
In ΔOBD, OB = OD = radii of same circle.
∴ ∠OBD =∠ODB
Thus, ∠OBD = 12 (180° – ∠BOD) = 12 (180° – 140°) = 12 × 40° = 20°

16. ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6 cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centres. Find the radii of the three circles.



(2011)

Answer:

Let the three radii be x, y, z respectively.

$x + y = 10$ .... (1)

$y + z = 8$ .... (2)

$x + z = 6$ .... (3)

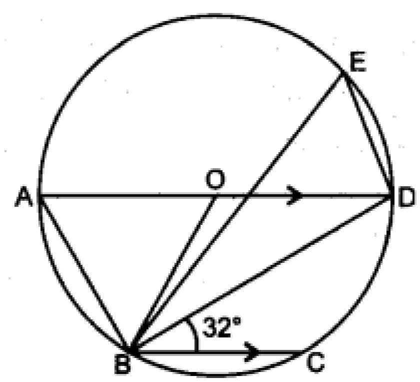
Adding equations (1), (2) and (3), $2x + 2y + 2z = 24$

$x + y + z = 12$ .... (4)

Subtracting each equation (1), (2) and (3) from equation (4), we get

$z = 2$ cm, $x = 4$ cm, $y = 6$ cm.

17. In the given figure below, AD is a diameter. O is the centre of the circle. AD is parallel to BC and $\angle CBD = 32°$.



Find :

(i) $\angle OBD$

(ii) $\angle AOB$

(iii) $\angle BED$ (2016)

Answer:

Given : AD ∥ BC

∴ $\angle ADB = \angle DBC = 32°$ (alternate angles)

∵ OB = OD (radii of the circle)

∴ (i) $\angle OBD = 32°$ (angles opposite to equal sides of a triangle)

(ii) $\angle AOB = 2\angle ODB$ (angle at the centre is twice the angle at remaining circumference)

$= 2 \times 32 = 64°$

(iii) In Δ ABD

$\angle ABD = 90°$ (angle in a semicircle)

$\angle ADB = 32°$

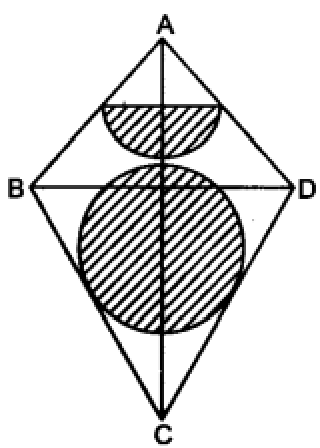
$\angle BAD = 180 - (90 + 32)$

$= 58°.$

$\angle BAD = \angle BED$ (angle in the same segment)

$\boxed{58° = \angle BED}$

18. The given figure represents a kite with a circular and a semicircular motif stuck on it. The radius of the circle is 2.5 cm and the semicircle is 2 cm. If diagonals AC and BD are the lengths 12 cm and 8 cm respectively, find the area of the :



(i) shaded part. Give your answer correct to the nearest whole number.
(ii) unshaded part.

(2016)

Answer:

(i) *Given* : Radius of circle = 2·5 cm, Radius of semicircle = 2 cm.

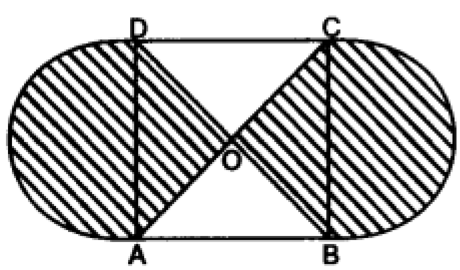
$$\text{Area of shaded part} = \text{Area of semicircle} + \text{Area of circle}$$
$$= \frac{1}{2}\pi r^2 + \pi r^2$$
$$= \frac{\pi(2)^2}{2} + \pi(2{\cdot}5)^2$$
$$= 2\pi + 6{\cdot}25\pi$$
$$= 8{\cdot}25\pi$$
$$= ₹\, 8{\cdot}25 \times \frac{22}{7}$$
$$= ₹\, 25{\cdot}92 \simeq 26 \text{ square cm.}$$

(ii) Let kite ABCD be a quadrilateral.

$$\therefore \quad \text{Area of quad.} = \frac{1}{2} \times \text{the product of the diagonals}$$
$$\because \quad \text{Area of kite} = \frac{1}{2}\text{BD} \times \text{AC}$$
$$= \frac{1}{2} \times 8 \times 12$$
$$= 48 \text{ square cm.}$$
$$\text{Area of unshaded part} = \text{Area of kite} - \text{Area of shaded part}$$
$$= 48 - 26$$

= 22 square cm

19. In the given figure, ABCD is a square of side 21 cm. AC and BD are two diagonals of the square. Two semi circles are drawn with AD and BC as diameters. Find the area of the shaded region.



(Take π = 22/7).

(2015)

Answer:

Given : Side = 21 cm,

Let Diagonal of the square = √2 (side)

$$\therefore \quad AC = BD = 21\sqrt{2}$$

$$\therefore \quad AO = OC = BO = OD = \frac{21\sqrt{2}}{2}$$

(Diagonals of square bisect each other)

$$\text{Area of } \Delta\, AOD = \text{Area of } \Delta\, BOC = \frac{1}{2} \times \frac{21\sqrt{2}}{2} \times \frac{21\sqrt{2}}{2} = \frac{441}{4} \text{ cm}^2.$$

$$\text{Area of semicircle} = \frac{1}{2}\pi r^2$$

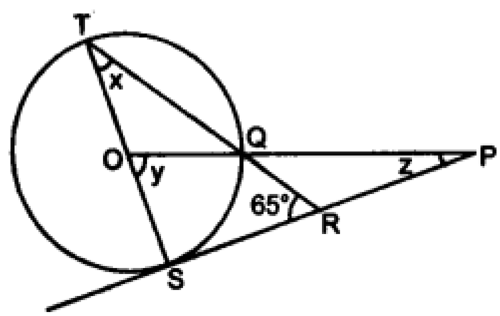
$$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^2 = \frac{693}{4} \text{ cm}^2.$$

Area of shaded region = Area of 2 semicircles + Area of Δ AOD + Area of Δ BOC

$$= 2 \times \frac{693}{4} + \frac{441}{4} + \frac{441}{4} = \frac{2268}{4}$$

$$= 567 \text{ cm}^2.$$

20. In the figure given below, O is the centre of the circle and SP is a tangent. If ∠SRT = 65°, find the value of x, y and z.



(2015)

Answer:

In Δ OSP, $\angle OSR = 90°$ [Radius is perpendicular to tangent]

In Δ TSR

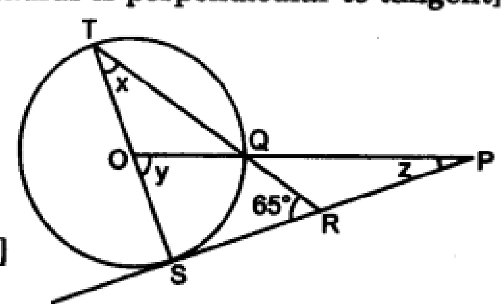
$$x + 90° + 65° = 180°$$

$$\therefore \quad x = 25°$$

$$\angle SOQ = 2\angle STR$$

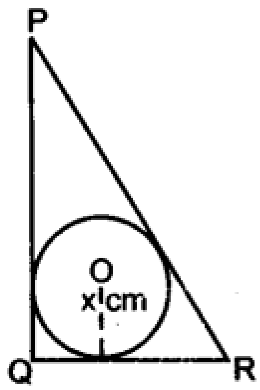
[Angle at centre = 2 × Angle at circumference]

$$y = 2 \times 25 = 50°$$



In Δ OSP,
50° + 90° +z = 180°
∴ z = 40°

21. In triangle PQR, PQ = 24 cm, QR = 7 cm and ∠PQR = 90°. Find the radius of the inscribed circle.



(2012)

Answer:

Given : Δ PQR is right angled.

$$PR^2 = PQ^2 + QR^2 = (24)^2 + (7)^2$$
$$= 576 + 49 = 625$$
$$\therefore \quad PR = 25 \text{ cm}$$

Draw ⊥r from O on PQ and PR and mark as B and C respectively.

$$\angle OBQ = \angle OAQ = \angle OCR$$
$$= 90°$$

(∠ between radius and tangent is 90°)

All ∠s of OAQB are 90° and QA = QB

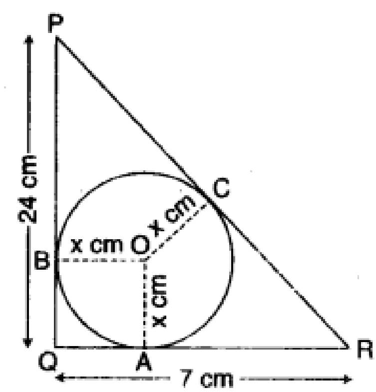
(Since the tangent to a circle from an exterior point are equal in length).

∴ OAQB is a square.

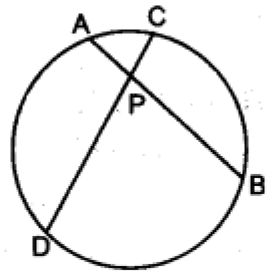
$$\therefore \quad QA = QB = x$$
$$AR = 7 - x = RC \quad (\because AR = RC)$$
$$BP = 12 - x = PC \quad (\because PB = PC)$$

Tangents from ext. point are equal

$$\because \quad PC + RC = PR$$
$$7 - x + 12 - x = 25$$
$$\therefore \quad x = 3 \text{ cm}$$

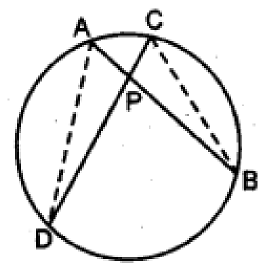


22. AB and CD are two chords of a circle intersecting at P. Prove that AP × PB = CP × PD.



(2015)

Answer:



To Prove : AP × PB = CP × PD

Construction : Join AD and CB.

Proof : In Δ APD and Δ CPB

$\angle A = \angle C$ [angles of the same segment]

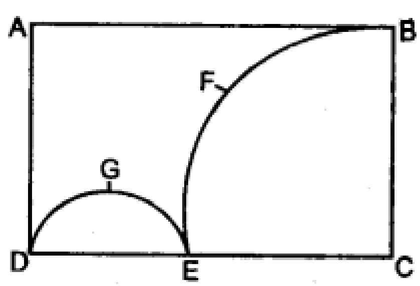
$\angle C = \angle B$ [angles of the same segment]

$\therefore$ Δ APD ~ Δ BPC [AA Postulate]

$\therefore$ $\frac{AP}{CP} = \frac{PD}{PB}$ [corresponding sides of similar Δs are proportional]

$\therefore$ AP × PB = CP × PD **Proved**

23. In the figure given below, ABCD is a rectangle. AB = 14 cm, BC = 7 cm. From the rectangle, a quarter circle BFEC and a semicircle DGE are removed.



Calculate the area of the remaining piece of the rectangle. (Take π = 22/7)

(2014)

Answer:

(c)

$$\text{Area of rectangle ABCD} = 14 \times 7 = 98 \text{ cm}^2$$

$$\text{Area of quarter circle BFEC} = \frac{1}{4}\pi(7)^2 = \frac{49}{4}\pi$$

$$\text{Area of semi-circle DGE} = \frac{1}{2}\pi\left(\frac{7}{2}\right)^2 = \frac{1}{2} \times \frac{49}{4}\pi$$

$$\text{Area of remaining piece of rectangle} = 98 - \left[\frac{49}{4}\pi + \frac{1}{2} \times \frac{49}{4}\pi\right]$$

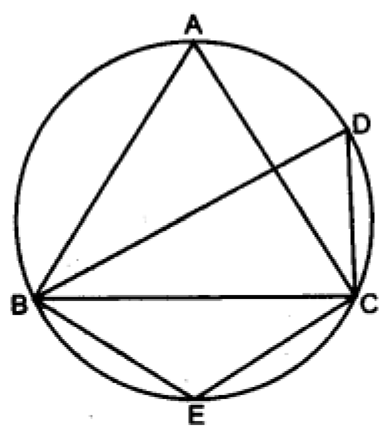
$$= 98 - \frac{49}{4}\pi\left[1 + \frac{1}{2}\right]$$

$$= 98 - \frac{49}{4} \times \frac{22}{7} \times \frac{3}{2} = 98 - \frac{231}{4}$$

$$= 98 - 57.75$$

$$= 40{\cdot}25 \text{ cm}^2.$$

24. In the figure, ∠DBC = 58°. BD is a diameter of the circle. Calculate :
(i) ∠BDC (ii) ∠BEC (iii) ∠BAC

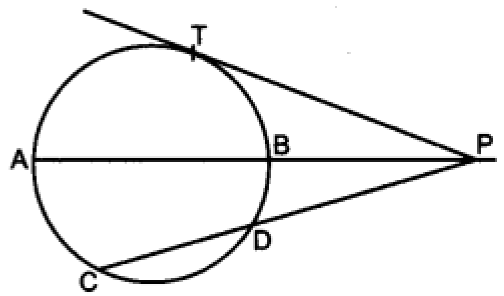


(2014)

Answer:
In Δ BCD; ∠DBC = 58°
(i) ∠BCD = 90° (Angle in the semicircle as BD is diameter)
∴ ∠DBC + ∠BCD + ∠BDC = 180°
⇒ 58°+ 90° +∠BDC = 180°
⇒ ∠BDC = 180° – (90° + 58°)
= 180°-148°
= 32° Ans.
(ii) ∠BEC + ∠BDC = 180° (∵BECD is a cyclic quadrilateral)

$\angle BEC = 180° - \angle BDC = 180° - 32°$
$\angle BEC = 148°$
(iii) $\angle BAC = \angle BDC$ (Angle of same segment are equal)
$\angle BAC = 32°$

25. In the figure given below, diameter AB and CD of a circle meet at P. PT is a tangent to the circle at T. CD=7.8 cm, PD=5 cm, PD=4 cm.



Find:
(i) AB.
(ii) the length of tangent PT. (2014)
ans.

(b) (i) Since chord CD and tangent at point T intersect each other at P,

$\therefore \quad PC \times PD = PT^2$

Since chord AB and tangent at point T intersect each other at P,

$\therefore \quad PA \times PB = PT^2$

From (1) and (2), $\quad PC \times PD = PA \times PB$

Given : CD = 7·8 cm; PD = 5 cm, PB = 4 cm.

$\therefore$ PA = PB + AB = 4 + AB, PC = PD + CD = 5 + 7·8 = 12·8 cm.

Putting these values in eq. (3)

$$12{\cdot}8 \times 5 = (4 + AB) \times 4$$

$$\Rightarrow \quad 4 + AB = \frac{12{\cdot}8 \times 5}{4}$$

$$\Rightarrow \quad 4 + AB = 16$$

$$\Rightarrow \quad AB = 12 \text{ cm.}$$

Hence, $\quad AB = 12$ cm.

(ii) From eq. (1), $\quad PT^2 = PA \times PB = 12{\cdot}8 \times 5$

$$\Rightarrow \quad PT^2 = 64$$

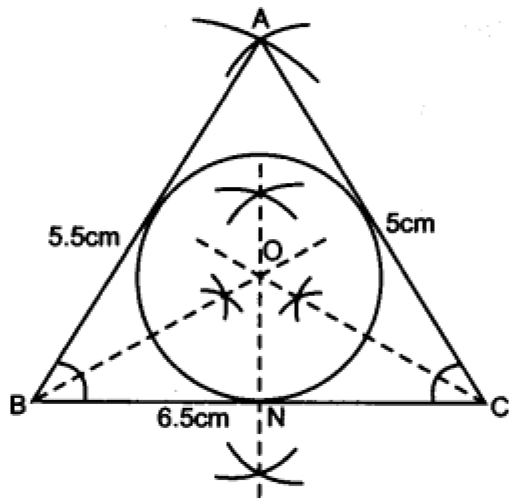
$$\Rightarrow \quad PT = 8 \text{ cm.} = \text{Length of tangent.}$$

26. Construct a Δ ABC with BC = 6.5 cm, AB = 5.5 cm, AC = 5 cm. Construct the incircle of the triangle. Measure and record the radius of the incircle.

(2014)

Answer:

Steps of construction:



(1) Construct a Δ ABC with the given data.

(2) Draw the internal bisectors of ∠B and ∠C. Let these bisectors cut at O.

(3) Taking O as centre. Draw a incircle which touches all the sides of the Δ ABC.

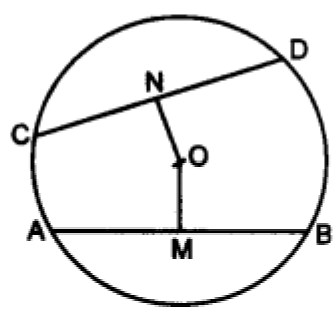
(4) From O draw a perpendicular to side BC which cut at N.

(5) Measure ON which is required radius of the incircle.

ON = 1.5 cm. (app.)

27. In the figure given below, O is the centre of the circle. AB and CD are two chords of the circle. OM is perpendicular to AB and ON is perpendicular to CD.

AB = 24 cm, OM = 5 cm, ON = 12 cm.



Find the :

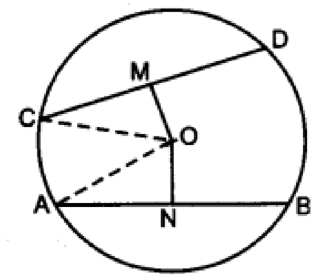
(i) radius of the circle.

(ii) length of chord CD.

(2014)

Answer:

(a) *Given* : AB = 24 cm; OM = 5 cm, ON = 12 cm.



$\because$ OM $\perp$ AB

M is mid point of AB.

$\therefore$ AM = 12 cm.

(i) Let radius of circle = r

From Δ AMO; $AO^2 = AM^2 + OM^2$ (By Pythagoras theorem)

$$r^2 = (12)^2 + (5)^2$$
$$= 144 + 25 = 169$$
$$r = 13 \text{ cm.}$$

(ii) Now from Δ CNO; $CO^2 = ON^2 + CN^2$

$$r^2 = (12)^2 + CN^2 \quad (\because AO = CO = r)$$
$$(13)^2 - (12)^2 = CN^2$$
$$169 - 144 = CN^2$$
$$\Rightarrow CN^2 = 25$$
$$\Rightarrow CN = 5$$

As ON $\perp$ CD, N is mid point of CD.

$\therefore$ CD = 2 CN = 2 × 5 = 10 cm.

28. **(b)** *In the given figure,*

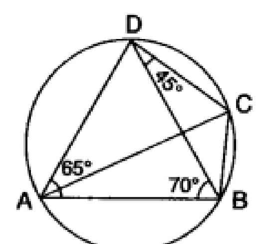
$\angle BAD = 65°$,

$\angle ABD = 70°$,

$\angle BDC = 45°$

(i) *Prove that AC is a diameter of the circle.*

(ii) *Find $\angle ACB$.*



[3]

(2013)

Answer:

(b) *Given* : $\angle BAD = 65°, \angle ABD = 70°, \angle BDC = 45°$

(i) $\because$ ABCD is a cyclic quadrilateral.

In Δ ABD,

$\angle BDA + \angle DAB + \angle ABD = 180°$ By using sum property of Δ^s

$\therefore \quad \angle BDA = 180° - (65° + 70°)$

$= 180° - 135°$

$= 45°$

Now from Δ ACD,

$\angle ADC = \angle ADB + \angle BDC$

$= 45° + 45°$ $\quad (\because \angle BDA = \angle ADB = 45°)$

$= 90°$

Hence, $\angle D$ makes right angle belongs in semi-circle therefore AC is a diameter of the circle.

(ii) $\angle ACB = \angle ADB$ (Angles in the same segment of a circle)

$\therefore \quad \angle ACB = 45°$ $\quad (\because \angle ADB = 45°)$

29. AB is a diameter of a circle with centre C = (-2, 5). If A = (3, -7). Find:
    (i) The length of radius AC
    (ii) The coordinates of B.

(2013)

Answer:

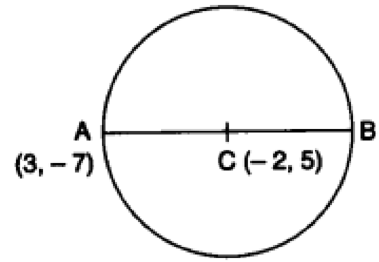
(i) The length of radius AC $= \sqrt{(-2-3)^2 + (5+7)^2}$

$= \sqrt{(-5)^2 + (12)^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13.$



(ii) Let the point of B be (x, y).

Given C is the mid-point of AB. Therefore

$$-2 = \frac{3+x}{2} \quad \text{and} \quad 5 = \frac{-7+y}{2}$$

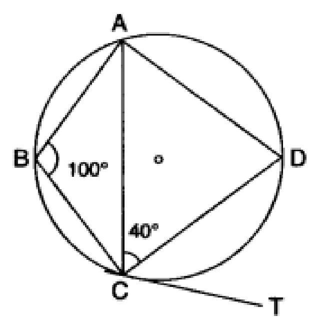
$\Rightarrow \quad 3 + x = -4 \qquad \Rightarrow \quad 10 = -7 + y$

$\Rightarrow \quad x = -4 - 3 = -7 \qquad y = 17$

Hence, the co-ordinate of B (–7, 17).

(b) *In the given circle with centre O, $\angle ABC = 100°$, $\angle ACD = 40°$ and CT is a tangent to the circle at C. Find $\angle ADC$ and $\angle DCT$.* **[3]**

30.



(2013)

Answer:

Given : $\angle ABC = 100°$

We know that,

$$\angle ABC + \angle ADC = 180° \quad \text{(The sum of opposite angles in a cyclic quadrilateral = 180°)}$$

$$\therefore \quad 100° + \angle ADC = 180°$$

$$\angle ADC = 180° - 100°$$

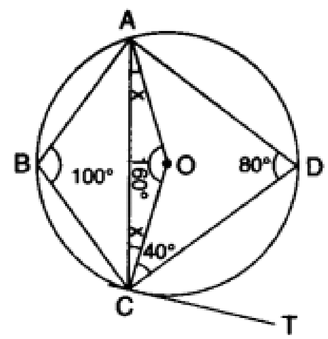
$$\angle ADC = 80°$$

Join OA and OC, we have a isosceles Δ OAC,

$$\because \quad OA = OC \quad \text{(Radii of a circle)}$$

$$\therefore \quad \angle AOC = 2 \times \angle ADC \quad \text{(by theorem)}$$

$$\text{or} \quad \angle AOC = 2 \times 80° = 160°$$



In Δ AOC,

$$\angle AOC + \angle OAC + \angle OCA = 180°$$

$$160° + \angle OCA + \angle OCA = 180° \quad [\because \angle OAC = \angle OCA]$$

$$2\angle OCA = 20°$$

$$\angle OCA = 10°$$

$$\angle OCA + \angle OCD = 40°$$

$$10° + \angle OCD = 40°$$

$$\therefore \quad \angle OCD = 30°$$

Hence, $\angle OCD + \angle DCT = \angle OCT$

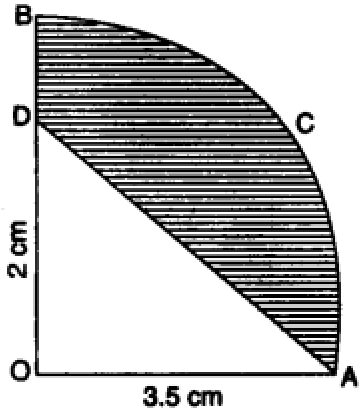
$$\because \quad \angle OCT = 90°$$

(The tangent at a point to circle is $\perp$ to the radius through the point to contant)

$$30° + \angle DCT = 90°$$

$$\therefore \quad \angle DCT = 60°$$

31. In the figure alongside, OAB is a quadrant of a circle. The radius OA = 3.5 cm and OD = 2 cm. Calculate the area of the shaded portion. (Take π = 22/7)



(2013)

Answer:

Radius of quadrant OACB = r = 3·5 cm

$\therefore$ Area of quadrant OACB $= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5 = 9{\cdot}625 \text{ cm}^2$

Here, $\angle AOD = 90°$

Then area of Δ AOD $= \frac{1}{2} \times \text{base} \times \text{height}$

Base = 3·5 cm and height = 2 cm

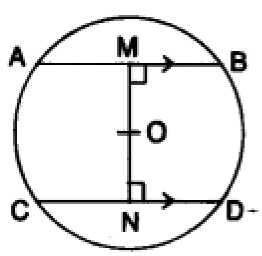
$\therefore$ $= \frac{1}{2} \times 3{\cdot}5 \times 2 = 3{\cdot}5 \text{ cm}^2$

Area of shaded portion = Area of quadrant – Area of triangle

$= 9{\cdot}625 - 3{\cdot}5$

$= 6{\cdot}125 \text{ cm}^2$

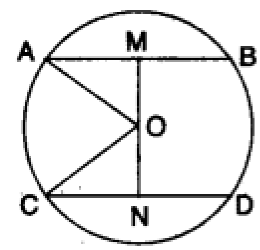
32. In the figure given alongside AB and CD are two parallel chords and O is the centre. If the radius of the circle is 15 cm, find the distance MN between the two chords of length 24 cm and 18 cm respectively.



(2010)

Answer:

Given : OA = OC = 15 cm, AB = 24 cm, CD = 18 cm.



Now $AM = 12, CN = 9$

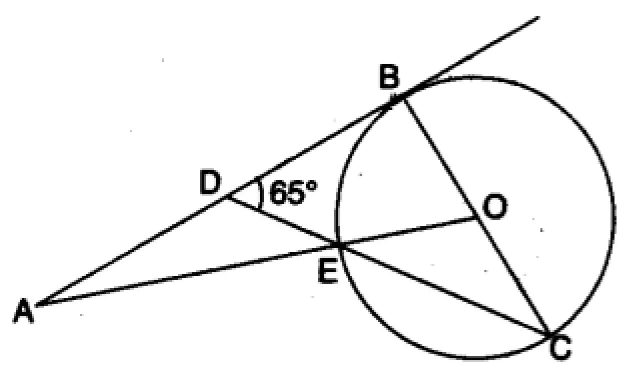
In Δ OAM,

$$OA^2 = OM^2 + AM^2$$
$$OM^2 = OA^2 - AM^2$$
$$= 15^2 - 12^2$$
$$= 225 - 144 = 81$$
$$OM = 9$$

Similarly, in Δ OCN,

$$OC^2 = ON^2 + CN^2$$
$$ON^2 = OC^2 - CN^2 = 15^2 - 9^2$$
$$= 225 - 81 = 144$$
$$ON = 12$$
$$MN = OM + ON = 9 + 12 = 21 \text{ cm.}$$

33. In the following figure O is the centre of the circle and AB is a tangent to it at point B. ∠BDC = 65°. Find ∠BAO.



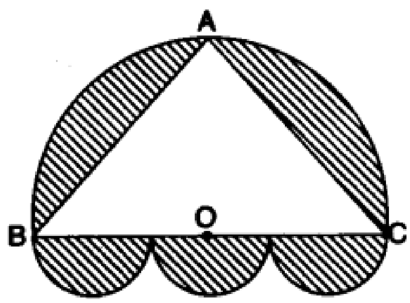
(2010)

Answer:

AB is tangent $\Rightarrow \angle ABO = 90°$

$$\angle BDC = 65° \text{ (given)}$$
$$\Rightarrow \quad \angle BCD = 90° - 65° = 25°$$
$$\angle BOE = 2 \times 25° \text{ (angle at centre)}$$
$$= 50°$$
$$\angle BAO = 90° - \angle BOE$$
$$\angle BAO = 90° - 50°$$
$$= 40°$$

34. A doorway is decorated as shown in the figure. There are four semi-circles. BC, the diameter of the larger semi circle is of length 84 cm. Centres of the three equal semi-circles lie on BC. ABC is an isosceles triangle with AB = AC. If BO = OC, find the area of the shaded region. (Take π = 22/7)



(2010)

Answer:

Let AB = AC = x cm.

As angle in semi circle is 90°

i.e., $\angle A = 90°$

In right angled Δ ABC, by Pythagoras theorem, we get

$$AB^2 + AC^2 = BC^2$$
$$x^2 + x^2 = 84^2$$

$$2x^2 = 84 \times 84$$

$$\therefore \quad x^2 = 84 \times 42$$

Now

$$\text{Area of } \Delta\, ABC = \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2} \times 84 \times 42$$

$$= 1764 \text{ cm}^2.$$

$$\text{Diameter of semicircle } (2r) = 84 \text{ cm}$$

$$\text{Radius } (r) = \frac{1}{2} \times 84 = 42 \text{ cm}$$

$$\therefore \quad \text{Area of semicircle} = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 42 \times 42$$

$$= 2772 \text{ cm}^2.$$

$$\text{Diameter of each (three equal) semicircles} = \frac{1}{3} \times 84 = 28 \text{ cm.}$$

$$\text{Radius of the 3 equal semicircles} = \frac{1}{2} \times 28 = 14 \text{ cm.}$$

$$\therefore \quad \text{Area of three equal semi circles} = 3 \times \frac{1}{2}\pi r^2$$

$$= 3 \times \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$$

$$= 924 \text{ cm}^2.$$

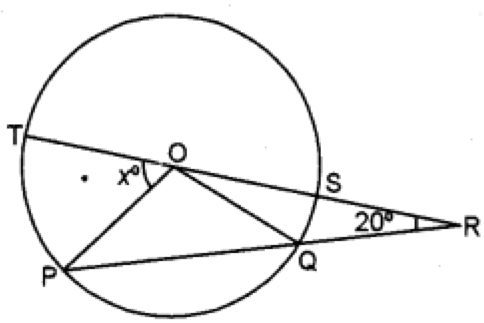
$$\text{Area of shaded region} = \text{Area of semicircles} + \text{Area of three equal circles} - \text{Area of } \Delta\, ABC$$

$$= 2772 + 924 - 1764$$

$$= 3696 - 1764$$

$$= 1932 \text{ cm}^2.$$

35. In the figure given below 'O' is the centre of the circle. If QR = OP and ∠ORP = 20°. Find the value of 'x ' giving reasons.



(2018)

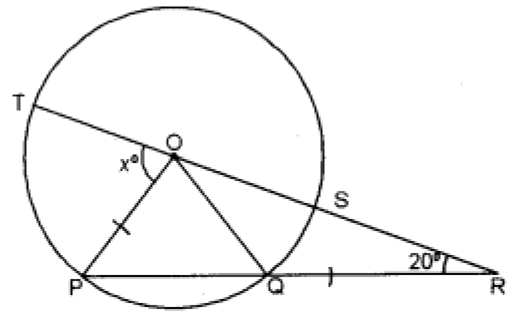
Answer:

Here, in ΔOPQ

OP = OQ = r

Also, OP = QR [Given]

OP = OQ = QR = r



In ΔOQR, OQ = QR

$\angle QOR = \angle ORP = 20°$

And $\angle OQP = \angle QOR + \angle ORQ$

$= 20° + 20°$

$= 40°$

Again, in Δ OPQ

$\angle POQ = 180° - \angle OPQ - \angle OQP$

$= 180° - 40° - 40°$

$= 100°$

Now, $x° + \angle POQ + \angle QOR = 180°$ [a straight angle]

$x° + 100° + 20° = 180°$

$x° = 180° - 120° = 60°$

Hence, the value of x is 60.